$$Y = \frac{\sigma}{\epsilon} = \frac{F}{A}\frac{l}{\Delta l} \tag{1}$$

$$\sigma = Y\epsilon \tag{2}$$

$$F = \frac{YA}{l}\Delta l \tag{3}$$

$$G = \frac{\tau}{\alpha} \tag{4}$$

$$\frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = -\nu\frac{\Delta l_1}{l_1} = -\nu\epsilon \tag{5}$$

$$(l_1 + \Delta l_1)(l_2 + \Delta l_2)(l_3 + \Delta l_3) \tag{6}$$

$$l_1 l_2 l_3 + \Delta l_1 l_2 l_3 + l_1 \Delta l_2 l_3 + l_1 l_2 \Delta l_3 \tag{7}$$

$$\begin{aligned} \Delta l_1 l_2 l_2 + l_1 \Delta l_2 l_3 + l_1 l_2 \Delta l_3 &= \Delta l_1 l_2 l_3 + l_1(-\nu\frac{\Delta l_1}{l_1} l_2) l_3 + l_1 l_2(-\nu\frac{\Delta l_1}{l_1} l_3) \\ &= (1 - 2\nu)\Delta l_1 l_2 l_3 \end{aligned} \tag{8}$$

$$G = \frac{Y}{2(1+\nu)} \tag{9}$$

$$\phi(x) = \phi(a) + \frac{d\phi(a)}{dx}(x-a) + \frac{1}{2}\frac{d^2\phi(a)}{dx^2}(x-a)^2 + \frac{1}{6}\frac{d^3\phi(a)}{dx^3}(x-a)^3 + \dots \tag{10}$$

$$\alpha = \tan^{-1}\left(\frac{x}{a}\right) \approx \frac{x}{a} \tag{11}$$

$$\tau = G\alpha \approx \frac{Gx}{a} \tag{12}$$

$$\tau = C\sin\left(\frac{2\pi x}{b}\right) \tag{13}$$

$$\frac{Gx}{a} \approx C\frac{2\pi x}{b} \tag{14}$$

$$\tau_Y = C = \frac{Gb}{2\pi a} = \frac{G}{\sqrt{3}\pi} \tag{15}$$