$$M\frac{d^2x}{dt^2} = -\gamma x \tag{1}$$

$$\omega = \sqrt{\frac{\gamma}{M}} \tag{2}$$

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}\gamma x^2 \tag{3}$$

$$\frac{1}{2}\gamma x_{\max}^2 = k_B T \tag{4}$$

$$x_{\max} = \left(\frac{2k_B T}{\gamma}\right)^{1/2} \tag{5}$$

$$M\frac{d^2u_n}{dt^2} = -\gamma(u_n - u_{n-1}) + \gamma(u_{n+1} - u_n) \tag{6}$$

$$M\frac{d^2u_n}{dt^2} = -\gamma[2u_n - u_{n-1} - u_{n+1}] \tag{7}$$

$$u_n(t) = ue^{i(kan-\omega t)} \tag{8}$$

$$-M\omega^2 e^{i(kan-\omega t)} = -\gamma[2 - e^{-ika} - e^{ika}]e^{i(kan-\omega t)} = -2\gamma(1-\cos ka)e^{i(kan-\omega t)} \tag{9}$$

$$\omega(k) = \sqrt{\frac{2\gamma(1-\cos ka)}{M}} = 2\sqrt{\frac{\gamma}{M}}|\sin\frac{ka}{2}| \tag{10}$$

$$\omega = \sqrt{\frac{\gamma}{M}}ak = \nu k \tag{11}$$

$$M_1\frac{d^2u_n}{dt^2} = -\gamma[2u_n - v_{n-1} - v_n] \qquad M_2\frac{d^2v_n}{dt^2} = -\gamma[2v_n - u_n - u_{n+1}] \tag{12}$$

$$u_n(t) = ue^{i(kbn-\omega t)} \qquad v_n(t) = ve^{i(kbn-\omega t)} \tag{13}$$

$$-\omega^2 M_1 u = \gamma v(1 + e^{-ikb}) - 2\gamma u \qquad -\omega^2 M_2 v = \gamma u(e^{ikb} + 1) - 2\gamma v \tag{14}$$

$$\begin{vmatrix} 2\gamma - \omega^2 M_1 & -\gamma(e^{ikb}+1) \\ -\gamma(1+e^{-ikb}) & 2\gamma - \omega^2 M_2 \end{vmatrix} = 0 \tag{15}$$

$$\omega^2 = \gamma\left(\frac{1}{M_1} + \frac{1}{M_2}\right) \pm \gamma\left[\left(\frac{1}{M_1} + \frac{1}{M_2}\right)^2 - \frac{4}{M_1 M_2}\sin^2\frac{kb}{2}\right]^{1/2} \tag{16}$$

$$u_{N+n}(t) = u_n(t) \tag{17}$$

$$e^{ikan} = e^{ika(N+n)} \tag{18}$$

so that

$$e^{ikNa} = 1 \tag{19}$$

$$k = \frac{2\pi}{aN}n \tag{20}$$

$$E_l = \left(l + \frac{1}{2}\right)\hbar\omega \tag{21}$$

$$E_l(k) = \left(l + \frac{1}{2}\right)\hbar\omega(k) \tag{22}$$

$$\mathbf{k} = (k_x, k_y, k_z) = \frac{2\pi}{aN}(n_x, n_y, n_z) = (\frac{n_x 2\pi}{L}, \frac{n_y 2\pi}{L}\frac{n_z 2\pi}{L}) \tag{23}$$

$$\sigma = \frac{F}{a^2} \tag{24}$$

$$F = \gamma x \tag{25}$$

$$\sigma = \frac{\gamma x}{a^2} \tag{26}$$

$$Y = \frac{\sigma}{\epsilon} = \frac{\gamma x}{a^2}\frac{a}{x} = \frac{\gamma}{a} \tag{27}$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_E \tag{28}$$

$$\langle E\rangle = 3N_A\left(\langle n\rangle + \frac{1}{2}\right)\hbar\omega_E \tag{29}$$

$$\langle n\rangle = \frac{1}{e^{\hbar\omega_E/k_BT} - 1} \tag{30}$$

$$\langle E\rangle = 3N_A\left(\frac{1}{e^{\hbar\omega_E/k_BT} - 1} + \frac{1}{2}\right)\hbar\omega_E \tag{31}$$

$$C = 3N_A\left(\frac{\partial\langle E\rangle}{\partial T}\right)_V = 3R\left(\frac{\hbar\omega_E}{k_BT}\right)^2\frac{e^{\hbar\omega_E/k_BT}}{(e^{\hbar\omega_E/k_BT} - 1)^2} \tag{32}$$

$$p_1 \propto e^{-\hbar\omega_E/k_BT} \tag{33}$$

$$\omega(k) = \nu k \tag{34}$$

$$\langle E\rangle = \frac{\hbar\omega}{e^{\hbar\omega/k_BT} - 1} \tag{35}$$

$$\langle E\rangle = 3\int \frac{\hbar\omega'}{e^{\hbar\omega'/k_BT} - 1}d\omega' \tag{36}$$

$$\langle E\rangle = 3\int_0^{\omega_D} \frac{g(\omega)\hbar\omega}{e^{\hbar\omega/k_BT} - 1}d\omega \tag{37}$$

$$N = \frac{4}{3}\pi n^3 \tag{38}$$

$$N = \frac{4}{3}\pi\left(\frac{L|\mathbf{k}|}{2\pi}\right)^3 \tag{39}$$

$$N(\omega) = \frac{4}{3}\pi\left(\frac{L\omega}{2\pi\nu}\right)^3 = \frac{V}{6\pi^2\nu^3}\omega^3 \tag{40}$$

$$g(\omega)d\omega = \frac{dN}{d\omega}d\omega = \frac{\omega^2 V}{2\pi^2\nu^3}d\omega \tag{41}$$

$$3N = 3\int_0^{\omega_D} g(\omega)d\omega \tag{42}$$

$$\omega_D^3 = 6\pi^2\frac{N}{V}\nu^3 \tag{43}$$

$$\langle E\rangle = 3\int_0^{\omega_D}\frac{\omega^2 V}{2\pi^2\nu^3}\frac{\hbar\omega}{e^{\hbar\omega/k_BT}-1}d\omega = \frac{3V\hbar}{2\pi^2\nu^3}\int_0^{\omega_D}\frac{\omega^3}{e^{\hbar\omega/k_BT}-1}d\omega \tag{44}$$

$$\langle E\rangle = \frac{3Vk_B^4T^4}{2\pi^2\nu^3\hbar^3}\int_0^{x_D}\frac{x^3}{e^x-1}dx = 9Nk_BT\left(\frac{T}{\Theta_D}\right)^3\int_0^{x_D}\frac{x^3}{e^x-1}dx \tag{45}$$

$$C = \frac{12\pi^4}{5}Nk_B\left(\frac{T}{\Theta_D}\right)^3 \tag{46}$$

$$\kappa = \kappa_p + \kappa_e \tag{47}$$

$$\kappa = \frac{1}{A}\frac{\partial Q}{\partial t}\frac{\Delta x}{\Delta T} \tag{48}$$

$$\kappa_p = \frac{1}{3}c\lambda_p v_p \tag{49}$$

$$\frac{\Delta L}{L} = \alpha\Delta T \tag{50}$$

$$G = U + PV - TS \tag{51}$$

$$T_m = \frac{(0.05a)^2\gamma}{2k_B} = \frac{(0.05a)^2\omega^2 M}{2k_B} \tag{52}$$

$$T_m = \frac{(0.05a)^2\Theta_D^2 k_B M}{2\hbar^2} \tag{53}$$