$$-\frac{\hbar^2\nabla^2}{2m_e}\psi(\mathbf{r})+U(\mathbf{r})\psi(\mathbf{r})=E\psi(\mathbf{r}) \tag{1}$$

$$U(\mathbf{r})=U(\mathbf{r}+\mathbf{R}), \tag{2}$$

$$\psi(\mathbf{r})=\psi(x,y,z)=\psi(x+lL,y+mL,z+nL), \tag{3}$$

$$\psi(\mathbf{r})=\frac{1}{\sqrt{V}}e^{i\mathbf{k}\cdot\mathbf{r}} \tag{4}$$

$$\mathbf{k}=(k_x,k_y,k_z)=(\frac{n_x 2\pi}{L},\frac{n_y 2\pi}{L}\frac{n_z 2\pi}{L}), \tag{5}$$

$$E(\mathbf{k})=\frac{\hbar^2k^2}{2m_e}=\frac{\hbar^2}{2m_e}(k_x^2+k_y^2+k_z^2) \tag{6}$$

$$\frac{\hbar^2}{2m_e}\left(\frac{2\pi}{L}\right)^2 \tag{7}$$

$$\frac{N}{2}=\frac{4}{3}\pi n_{\max}^3, \tag{8}$$

$$n_{\max}=\left(\frac{3N}{8\pi}\right)^{1/3} \tag{9}$$

$$E_{\max}=\frac{\hbar^2k_{\max}^2}{2m_e}=\frac{\hbar^2}{2m_e}\left(\frac{2\pi}{L}\right)^2 n_{\max}^2 \tag{10}$$

$$E_F=\frac{\hbar^2}{2m_e}\left(3\pi^2 n\right)^{2/3} \tag{11}$$

$$E(N)=\frac{\hbar^2}{2m_e}\left(\frac{3\pi^2N}{V}\right)^{2/3}, \tag{12}$$

$$g(E)dE=\frac{dN}{dE}dE=\frac{V}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}E^{1/2}dE \tag{13}$$

$$f(E,T)=\frac{1}{e^{(E-\mu)/k_BT}+1}, \tag{14}$$

$$N=\int_0^\infty g(E)f(E,T)dE \tag{15}$$

$$E=\frac{3}{2}k_BTg(E_F)k_BT \tag{16}$$

$$C=\left(\frac{\partial E}{\partial T}\right)_V=3k_B^2Tg(E_F) \tag{17}$$

$$C=\frac{\pi^2}{3}k_B^2Tg(E_F) \tag{18}$$

$$\frac{\kappa}{\sigma}=\frac{\pi^2}{3}\frac{k_B^2}{e^2}T=LT \tag{19}$$

$$\phi_0(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}, \tag{20}$$

$$\phi(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}e^{-r/r_{\mathrm{TF}}}, \tag{21}$$

$$r_{\mathrm{TF}} = \sqrt{\frac{V\epsilon_0}{e^2 g(E_F)}} \tag{22}$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}u_{\mathbf{k}}(\mathbf{r}), \tag{23}$$

$$u_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r}+\mathbf{R}) \tag{24}$$

$$\psi(\mathbf{r}) = \sum_{\mathbf{k}} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{25}$$

$$U(\mathbf{r}) = \sum_{\mathbf{G}} U_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{r}} \tag{26}$$

$$U_{-\mathbf{G}} = U^*_{\mathbf{G}} \tag{27}$$

$$-\frac{\hbar^2\nabla^2}{2m_e}\psi(\mathbf{r}) = \sum_{\mathbf{k}} \frac{\hbar^2 k^2}{2m_e} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \tag{28}$$

$$U(\mathbf{r})\psi(\mathbf{r}) = \left(\sum_{\mathbf{G}} U_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{r}}\right)\left(\sum_{\mathbf{k}} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\right) = \sum_{\mathbf{kG}} U_{\mathbf{G}} c_{\mathbf{k}} e^{i(\mathbf{G}+\mathbf{k})\mathbf{r}} = \sum_{\mathbf{k'G}} U_{\mathbf{G}} c_{\mathbf{k'}-\mathbf{G}} e^{i\mathbf{k'}\cdot\mathbf{r}} \tag{29}$$

$$\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\left\{\left(\frac{\hbar^2 k^2}{2m_e} - E\right)c_{\mathbf{k}} + \sum_{\mathbf{G}} U_{\mathbf{G}} c_{\mathbf{k}-\mathbf{G}}\right\} = 0 \tag{30}$$

$$\left(\frac{\hbar^2 k^2}{2m_e} - E\right)c_{\mathbf{k}} + \sum_{\mathbf{G}} U_{\mathbf{G}} c_{\mathbf{k}-\mathbf{G}} = 0 \tag{31}$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = \sum_{\mathbf{G}} c_{\mathbf{k}-\mathbf{G}} e^{i(\mathbf{k}-\mathbf{G})\cdot\mathbf{r}} \tag{32}$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{kr}}\left(\sum_{\mathbf{G}} c_{\mathbf{k}-\mathbf{G}} e^{-i\mathbf{G}\cdot\mathbf{r}}\right) \tag{33}$$

$$\psi_{\mathbf{k}+\mathbf{G'}}(\mathbf{r}) = \sum_{\mathbf{G}+\mathbf{G'}} c_{\mathbf{k}-\mathbf{G}+\mathbf{G'}} e^{i(\mathbf{k}-\mathbf{G}+\mathbf{G'})\cdot\mathbf{r}}, \tag{34}$$

$$\psi_{\mathbf{k}+\mathbf{G'}} = \psi_{\mathbf{k}}(\mathbf{r}) \tag{35}$$

$$E(\mathbf{k}+\mathbf{G'}) = E(\mathbf{k}) \tag{36}$$

$$U(x) = \sum_{n} U_n e^{ingx}, \tag{37}$$

$$\left(\frac{\hbar^2(k-g)^2}{2m_e} - E\right)c_{k-g} + Uc_k = 0$$
$$\left(\frac{\hbar^2 k^2}{2m_e} - E\right)c_k + Uc_{k-g} + Uc_{k+g} = 0$$
$$\left(\frac{\hbar^2(k+g)^2}{2m_e} - E\right)c_{k+g} + Uc_k = 0 \tag{38}$$

$$-i\hbar\nabla\psi_{\mathbf{k}}(\mathbf{r}) = \hbar\mathbf{k}\psi_{\mathbf{k}}(\mathbf{r}) - e^{i\mathbf{k}\cdot\mathbf{r}}i\hbar\nabla u_{\mathbf{k}}(\mathbf{r}), \tag{39}$$

$$\psi(+) \propto e^{i\frac{\pi}{a}x} + e^{-i\frac{\pi}{a}x} = 2\cos(\frac{\pi}{a}x) \tag{40}$$
$$\psi(-) \propto e^{i\frac{\pi}{a}x} - e^{-i\frac{\pi}{a}x} = 2i\sin(\frac{\pi}{a}x) \tag{41}$$

$$v_g = \frac{d\omega(k)}{dk} = \frac{1}{\hbar}\frac{dE(k)}{dk} \tag{42}$$
$$dE = -e\mathcal{E}v_g dt \tag{43}$$
$$\frac{dE}{dt} = \frac{dE}{dk}\frac{dk}{dt}, \tag{44}$$
$$\hbar\frac{dk}{dt} = -e\mathcal{E} \tag{45}$$
$$a = \frac{dv_g}{dt} = \frac{1}{\hbar}\frac{d}{dt}\frac{dE(k)}{dk} = \frac{1}{\hbar}\frac{d^2E(k)}{dk^2}\frac{dk}{dt} \tag{46}$$
$$a = -\frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2}e\mathcal{E} \tag{47}$$
$$m^* = \hbar^2\left(\frac{d^2E(k)}{dk^2}\right)^{-1} \tag{48}$$