$$\oint \mathbf{B} d\mathbf{A} = 0 \qquad \mathrm{div}\mathbf{B} = 0 \tag{1}$$

$$\mathbf{B} = \mu_0 \mathbf{H} \tag{2}$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mathbf{B}_0 + \mu_0 \mathbf{M} \tag{3}$$

$$\mathbf{M} = \mu \frac{N}{V} \tag{4}$$

$$\mu_0 \mathbf{M} = \chi_{\mathrm{m}} \mathbf{B}_0 \tag{5}$$

$$U = -V \int_0^{B_0} M dB'_0 = -V \int_0^{B_0} \frac{\chi_{\mathrm{m}}}{\mu_0} B'_0 dB'_0 = -V \frac{\chi_{\mathrm{m}}}{2\mu_0} B_0^2 \tag{6}$$

$$|\mu| = \frac{e\hbar\sqrt{l(l+1)}}{2m_e} = \sqrt{l(l+1)}\mu_B \tag{7}$$

$$\mu_l = -\frac{e\hbar m_l}{2m_e} = -m_l \mu_B \tag{8}$$

$$\mu_J = -g m_J \mu_B \tag{9}$$

$$L = \sum m_l \quad S = \sum m_s \tag{10}$$

$$g = \frac{3J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \tag{11}$$

$$\mathbf{B} = \mathrm{curl}\mathbf{A} \tag{12}$$

$$\mathbf{p} \to \mathbf{p} - q\mathbf{A} \tag{13}$$

$$\mathbf{A} = -\frac{1}{2} \mathbf{r} \times \mathbf{B}_0 \tag{14}$$

$$H_{\mathrm{kin}} \to H'_{\mathrm{kin}}$$
$$\tfrac{\mathbf{p}^2}{2m_e} \to \tfrac{1}{2m_e}\left(\mathbf{p} + e\mathbf{A}\right)^2 = \tfrac{1}{2m_e}\left(\mathbf{p} - e\tfrac{\mathbf{r}\times\mathbf{B}_0}{2}\right)^2$$

$$H'_{\mathrm{kin}} = \frac{1}{2m_e}\left(\mathbf{p}^2 + e\mathbf{B}_0 \cdot (\mathbf{r} \times \mathbf{p}) + \frac{e^2}{4}(\mathbf{r} \times \mathbf{B}_0)^2\right) \tag{15}$$

$$H'_{\mathrm{kin}} = H_{\mathrm{kin}} + H' = \frac{\mathbf{p}^2}{2m_e} + \frac{e}{2m_e} B_0 (\mathbf{r} \times \mathbf{p})_z + \frac{e^2}{8m_e} B_0^2 (x^2 + y^2) \tag{16}$$

$$E' = \frac{e}{2m_e} B_0 \langle\psi|(\mathbf{r} \times \mathbf{p})_z|\psi\rangle + \frac{e^2}{8m_e} B_0^2 \langle\psi|(x^2 + y^2)|\psi\rangle \tag{17}$$

$$\mu = -\frac{\partial E'}{\partial B_0} = -\frac{e^2}{4m_e} B_0 \langle\psi|(x^2 + y^2)|\psi\rangle \tag{18}$$

$$\mu = -\frac{Ze^2}{6m_e} r_a^2 B_0 \tag{19}$$

$$\chi_{\mathrm{m}} = \mu_0 \frac{M}{B_0} = -\frac{\mu_0 Z N e^2}{6 V m_e} r_a^2 \tag{20}$$

$$\chi_{\mathrm{m}} = -\frac{1}{3V} \mu_B^2 \mu_0 g(E_F) \left(\frac{m_e}{m^*} \right)^2 \tag{21}$$

$$Z = \sum_{m_J=-J}^{J} e^{\frac{-g\mu_B m_J B_0}{k_B T}} \tag{22}$$

$$\bar{\mu}_C = \frac{1}{Z} \sum_{m_J=-J}^{J} g\mu_B m_J e^{\frac{g\mu_B m_J B_0}{k_B T}} \tag{23}$$

$$\chi_C = \frac{C}{T} \tag{24}$$

$$C = \frac{\mu_0 N g^2 \mu_B^2 J(J+1)}{3V k_B} \tag{25}$$

$$N_{\downarrow\downarrow B_0} - N_{\downarrow\uparrow B_0} = g(E_F)\mu_B B_0 \tag{26}$$

$$M = \frac{1}{V}(N_{\downarrow\downarrow B_0} - N_{\downarrow\uparrow B_0})\mu_B = \frac{1}{V} g(E_F)\mu_B^2 B_0 \tag{27}$$

$$\chi_{\mathrm{m}} = \frac{1}{V} \mu_0 \mu_B^2 g(E_F) \tag{28}$$

$$E_{\uparrow\uparrow} - E_{\uparrow\downarrow} = -2X \tag{29}$$

$$E = -\mathbf{S}_i \cdot \sum_j 2X_{ij} \mathbf{S}_j \tag{30}$$

$$\begin{aligned} E_\uparrow(\mathbf{k}) &= E(\mathbf{k}) - I\frac{(n_\uparrow - n_\downarrow)}{N} \\ E_\downarrow(\mathbf{k}) &= E(\mathbf{k}) + I\frac{(n_\uparrow - n_\downarrow)}{N} \end{aligned} \tag{31}$$

$$R = \frac{n_\uparrow - n_\downarrow}{N} \tag{32}$$

$$\begin{aligned} n_\uparrow &= N \sum_{\mathbf{k}} f(E_\uparrow(\mathbf{k}), T) \\ n_\downarrow &= N \sum_{\mathbf{k}} f(E_\downarrow(\mathbf{k}), T) \end{aligned} \tag{33}$$

$$R = \frac{1}{N}(n_\uparrow - n_\downarrow) = \sum_{\mathbf{k}} \left(f(E_\uparrow(\mathbf{k}), T) - f(E_\downarrow(\mathbf{k}), T)\right)$$
$$= \sum_{\mathbf{k}} \left(\frac{1}{e^{(E_\uparrow(\mathbf{k}) - IR - E_F)/k_B T} + 1} - \frac{1}{e^{(E_\downarrow(\mathbf{k}) + IR - E_F)/k_B T} + 1} \right) \tag{34}$$

$$\chi_{\mathrm{m}} = \frac{C}{T - \Theta_c} \tag{35}$$