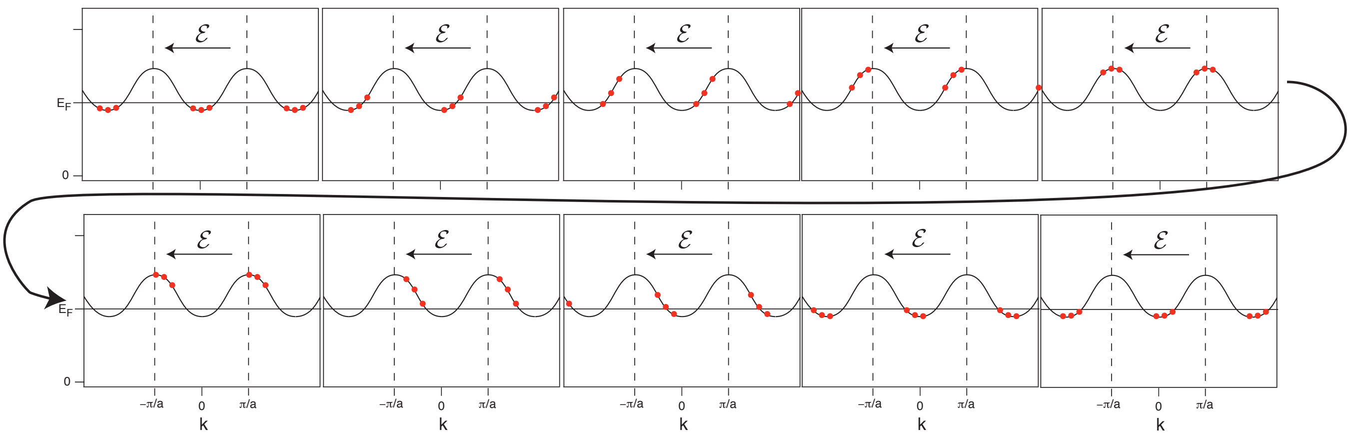

Figure 1: Illustration of Bloch oscillations. Starting in the left upper image, the series illustrates what happens to the electrons below the Fermi level in a metal when a DC electric field is turned on and time passes without any scattering events.

Bloch Oscillations

Considering the simple picture of conduction given by Figure 6.16 in the book and the "equation of motion" in reciprocal space

$$\hbar \frac{dk}{dt} = -e\mathcal{E}, \tag{1}$$

what should we expect for the conductivity in a metal if we turn all possible scattering processes off? In Figure 6.16, the electrons will move to ever higher k values, increasing their group velocity in the direction opposite to the field. This will give rise to an unlimited rise of the current density and an infinite conductivity.

While the entire exercise of imagining a situation without any scattering is admittedly a bit artificial, an infinite conductivity would only be expected in a free electron model. As soon as lattice periodicity is introduced, something even more odd happens and this phenomenon is called "Bloch oscillation". Fig. 1 shows the same scenario as Figure 6.16 but now for a lattice periodic potential. There is only one band shown that has the required periodicity in k. This band is partially filled such that the one dimensional solid is a metal.

We imagine a static electric field turned on the the uppermost left image and then observe the electrons as time passes. According to (1), the electrons increase their k at a constant rate. Now, however, this does no longer mean that they also increase their group velocity. This happens only initially. At some point, they have reached the maximum in the dispersion, corresponding to a group velocity of zero. A little later (in the second row of images), the move to lower energies again and in the process acquire a negative group velocity, moving in the field direction and not opposite to it. If this happens, we would not observe infinite conductivity but rather an AC current that is induced by an applied DC voltage.

The frequency of the AC current must be related to the electric field and the size of the reciprocal lattice. We can calculate the frequency because we can determine how long it takes to move the electrons by one Brillouin zone

$$\Delta k = \frac{2\pi}{a} = \frac{e\mathcal{E}\tau_B}{\hbar}.$$

Consequently,

$$\tau_B = \frac{h}{e\mathcal{E}a}$$

and

$$\omega_B = \frac{e\mathcal{E}a}{\hbar}$$

Is it at all possible to observe this? The effect can only be seen if τ_B is shorter than the usual relaxation time τ. Inserting numbers for a "normal" situation in a metal results in $\tau_B \approx 10^{-11}$ s which is substantially longer than a typical (room temperature) relaxation time of $\tau \approx 10^{-14}$ s. But there are some possibilities to make the Bloch oscillations observable. The first is to go to low temperatures and almost perfect solids in order to increase τ with respect to τ_B. The second is to artificially change the lattice constant a by using a free-electron like material and building an artificial lattice with an a that is much bigger than the atomic-scale a (for example using some nano-structuring technique). This will then lead to a reduced distance in reciprocal space the electrons have to travel to complete one oscillation such that the effect can be observed.